



FURTHER MATHEMATICS

9795/02

Paper 2 Further Application of Mathematics

May/June 2017

MARK SCHEME

Maximum Mark: 120

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE[®], Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

© IGCSE is a registered trademark.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **9** printed pages.

Question	Answer	Marks	Notes
1(i)	On average 95% of all identically constructed confidence intervals contain the parameter	B1	Use of “confident” without explanation: B0.
1(ii)	$\bar{x} = 39.72$	B1	
	$s_{n-1} = 3.30711$	B1	3.31 or 10.9
	$39.72 \pm 2.776 \times \frac{3.30711}{\sqrt{5}}$	M1	Needs $\sqrt{5}$ but allow s_n , z , or FT errors
		A1	All numbers correct (apart from s_{n-1}) soi
	= awrt (35.6, 43.8)	A1	Both correct to 3 SF. Condone wrong order.
2(i)	$\frac{1}{3}t^{-1} + \frac{2}{3}t^2$	B2	B1 for each If x used in an otherwise correct expression then SC1. Condone further consistent use in part (ii).
2(ii)(a)	$G_{10}(t) = (\frac{1}{3}t^{-1} + \frac{2}{3}t^2)^{10}$ oe	M1	$[G(t)]^{10}$
	Differentiate (could be any wrong G) and put $t = 1$	M1	
	$G'_{10}(t) = 10(-\frac{1}{3}t^{-2} + \frac{4}{3}t)(\frac{1}{3}t^{-1} + \frac{2}{3}t^2)^9$	A1	Correct derivative in any form
	$G'(1) = 10$	A1	Answer 10 only. www SC: $E(X) = 1$ B1, $E(T) = 10$ “E(X)” B1ft. Max 2/4
2(ii)(b)	Attempt coefficient of t^8	M1	
	Coefficient of $t^8 = {}^{10}C_4 (\frac{1}{3})^4 (\frac{2}{3})^6$	A1	Correct expression (term or coefficient)
	= 4480/19683 or awrt 0.228	A1	SC: 6 2s and 4 –1s: ${}^{10}C_4 (\frac{1}{3})^4 (\frac{2}{3})^6$ M1 0.228 A1 (Max 2/3)
3(i)	$\frac{32}{100} \pm 2.576 \sqrt{\frac{32}{100} \times \frac{68}{100} \div 100}$	M1	Correct form incl. 100. Allow $n/(n-1)$ for M1.
		A1	2.576
	= awrt (0.200, 0.440)	A1	Answer, limits correct to ≥ 3 sf. Condone wrong order
3(ii)	$2 \times 2.576 \times \sqrt{\frac{0.32 \times 0.68}{n}} = 0.04$ o.e.	M1	Correct equation, allow same wrong z or σ^2 as in (i), or 2 omitted. Allow $n-1$ for M1.
		M1	Solve including squaring
	$n_{\min} = 3610$	A1	3610 only

Question	Answer	Marks	Notes
4(i)	Use total area = 1 with integration	M1	
	$a + a\left[1 - \frac{1}{3}\right] = 1$	A1	Correct integration
	$a = \frac{3}{5}$	A1	0.6 or $\frac{3}{5}$ only
4(ii)	Integrate for one non-zero region (a can remain, constants can be omitted)	M1	
	$\begin{cases} \frac{3}{5}(x+1) & -1 \leq x < 0 \\ \frac{3}{5}\left(x - \frac{x^3}{3} + 1\right) & 0 \leq x \leq 1 \\ 0 & x < -1 \\ 1 & x > 1 \end{cases}$	4	A1 One formula correct A1 Other formula correct A1 Ranges $-1 \leq x < 0$ and $0 \leq x \leq 1$ correct, allow $\leq / <$ B1 0 and 1
4(iii)	$F(0.25) = 0.746875 < 0.75$	M1	Evaluate $F(0.25)$ and compare with 0.75
	$UQ > 0.25$	A1	Correct conclusion from correct values Alt method by finding UQ: $4u^3 - 12u + 3 = 0$ (oe) M1 Correct solution (e.g. by GC $u = 0.25556\dots$) (or sign change to demonstrate root between 0.25 and 1) and correct conclusion A1 Alt method: $u = 0.25 + u^3/3 > 0.25$ and conclusion B2
5(i)	Po(4)	M1	Po(4) and “1 –” in tables, e.g. 0.2149 or 0.0511
	$1 - P(\leq 6) = \text{awrt } 0.111$	A1	
5(ii)	$e^{-\lambda} = 0.6$	M1	(λ could be $t/180$ or $t/3$ or $20t$ depending on units)
	$\lambda = -\ln 0.6$ or awrt 0.511	A1	soi
	$t = 92$ seconds	A1	Answer 92 seconds or 1 minute 32 seconds only
5(iii)	$Po(60) \approx N(60, 60)$	M1A1	Po(60) and $N(60, \dots)$; $N(60, 60)$ soi
		M1	Standardise
		A1	Correct $\sqrt{\quad}$ and cc
	$\Phi\left(\frac{65.5 - 60}{\sqrt{60}}\right) = \Phi(0.71) = \text{awrt } 0.761$	A1	Answer, a.r.t. 0.761 [no cc, 0.7406; wrong cc, 0.7193; 60 not $\sqrt{60}$, 0.5366]

Question	Answer	Marks	Notes
6(i)	Attempt $\int f(x)e^{xt} dx$, correct limits (could be later)	M1	
	$\int_{-1}^1 f(x)e^{xt} dx = \int_{-1}^1 \frac{1}{2} e^{xt} dx$	A1	Correct expression
	$= \frac{1}{2t} [e^{xt}]_{-1}^1$ AG	A1	Correct integral
	$= \frac{e^t - e^{-t}}{2t} = \frac{\sinh t}{t}$	A1	Correctly obtain given answer SC Using formula for MGF of uniform distribution from formula book. Use of formula and substituting $a = -1$, $b = 1$ M1 Correctly obtain given answer A1 Max 2/4
6(ii)	$\frac{1}{t} \left(t + \frac{t^3}{3!} + \frac{t^5}{5!} + \dots \right)$	M1	Correct expansion of $\sinh t$ used
	$= 1 + \frac{t^2}{6} + \frac{t^4}{120} + \dots$	A1	Correct after division by t (at least 3 terms) soi
	$E(X^2) = M''_X(0) = 2! \times \text{coeff of } t^2 = \frac{1}{3}$	M1	Use $2! \times \text{coeff of } t^2$ or attempt to diff twice
	$\text{Var}(X) = \frac{1}{3}$	A1	$\frac{1}{3}$, distinction between $E(X^2)$ and $\text{Var}(X)$ made,
	$E(X^4) = 4! \times \text{coeff of } t^4 = \frac{1}{5}$	A1	or $E(X)$ (or $M'_X(0)$) stated to be zero Correctly obtain 1/5
7(i)	N(570, ...	M1	
	... 470)	A1	
	$1 - \Phi(1.845) = 0.0325$	A1	

Question	Answer	Marks	Notes
7(ii)	$P(C > (P + J + C)/8)$	M1A1	or equivalent e.g. $\frac{7}{8}C - \frac{1}{8}P - \frac{1}{8}J \sim N(-\frac{5}{4}, \frac{475}{32})$
	$= P(7C - P - J > 0)$	M1	
	$7C - P - J \sim N(-10, 950)$	M1A1	SC: Common error: forgetting that $T (= P + J + C)$ and C are not independent
	$P(> 0) = 1 - \Phi((0 - (-10))/\sqrt{950})$	M1	
	$= 1 - \Phi(0.3244) = \text{awrt } 0.373$	A1	If $Y = k(8C - T)$ then $Y \sim N(-10k, 1110k^2)$ for non-zero k . Allow M1 A1. Then $P(Y > 0)$ (or $P(Y < 0)$ if $k < 0$) = $= 1 - \Phi((0 - (-10))/\sqrt{1110}) = 1 - \Phi(0.3002)$ M1dep = awrt 0.382 A1 Max 4/7
8(i)	$\dot{\omega} = 4.2$	B1	
8(ii)	$r\dot{\omega} = 0.63$	B1	$r\dot{\omega}$ seen
	$r\omega^2 = \pm 0.216$	B1	$r\omega^2$ seen
	$ a = \sqrt{(0.63^2 + 0.216^2)}$	M1	Find acceleration for stated components
	$= 0.666$	A1	Answer, a.r.t. 0.666
9(i)	Trajectory formula quoted or obtained	M1	
	$y = x \tan \theta - \frac{x^2(1 + \tan^2 \theta)}{320}$	A1	Correct including $1 + \tan^2 \theta$, can be recovered (condone g/3200 in second term)
9(ii)	Substitute	M1	
	$y = 72 \tan \theta - \frac{72^2}{320}(1 + \tan^2 \theta)$	A1	
	$\alpha : 16.2t^2 - 72t + (16.2 + y) = 0$	M1	β : Differentiate w.r.t. θ
	$b^2 = 4ac \Rightarrow 72^2 = 4 \times 16.2(y + 16.2)$	M1	$72 \sec^2 \theta - 32.4 \tan \theta \sec^2 \theta = 0$
	$\Rightarrow y = 63.8$	A1	Condone $y \leq [\tan \theta = 20/9]$, $y = 63.8$ [exact] Alt: Completing the square or using $-b/2a = 20/9$
10(i)	$WD = P \times t = 75 \times 60$	M1	
	$= 4500 \text{ kJ or } 4500000 \text{ J oe}$	A1	Allow 4500000 for A1 but not 4500

Question	Answer	Marks	Notes
10(ii)	$v = 24 \text{ ms}^{-1}$ so $F = P/v$	M1*	Find resistive force (could be found in (i))
	$= 3125$	A1	3125 seen
		M1*	N2 with P/v (used here or later) and component of weight
	$\frac{75000}{v} - 0.05 \times 4000g - 3125 = 0$	A1	Correct equation
		M1dep*	Solve for v
	Hence $v = 14.6 \text{ ms}^{-1}$ $75000 = 0.05v \times 4000g + 3125v$ $75000 = 5125v$	A1	Answer, in range [14.6, 14.7] Alternative approach to N2 equation considering energy instead of force: (if 1s considered but could be multiplied throughout by eg 60). M1 for energy balance equation involving only PE, work done against resistance and work done by engine A1 correct
11(i)	Moments about A :	M1	Take moments about A involving a component of P and weight. Must be force \times distance.
	$1.25 \times P \sin \alpha = 0.4 \times 1.6g$	A1	Correct equation, $P \sin \alpha$ needs deriving
	$P \sin \alpha = 5.12 \text{ AG}$	A1	Correctly obtain given answer
11(ii)	N2(\uparrow): $P \sin \alpha + N = 1.6g$	M1A1	3 forces with a component of P .
	N2(\rightarrow): $P \cos \alpha = F$	B1	Correct equation (soi)
	$F \leq \frac{6}{17}N$ or 3.84	M1	Use $F \leq \mu N$ or $F = \mu N$
	$N = 10.88$, $P \cos \alpha \leq 3.84$ $P^2(\sin^2 \alpha + \cos^2 \alpha) \leq 40.96$	M1	Value for $P \cos \alpha$ and eliminate α (allow from $\alpha = 53.1^\circ$)
	$P \leq 6.4 \text{ AG}$	A1	Correctly obtain AG, inequalities correct (NB $\tan \alpha \geq 4/3$) throughout (or convincing argument for changing equation to inequality www)
12(i)	$0.6v \frac{dv}{dx} = -10 \times 0.6 - 0.024v^2$	M1	Use $v \, dv/dx$ for a and two other terms in $F = ma$
	Hence $v \frac{dv}{dx} = -10 - 0.04v^2$	A1	AG, completely correct, signs not fudged

Question	Answer	Marks	Notes
12(ii)	$\int \frac{v}{10 + 0.04v^2} dv = -\int dx$	M1	Separate variables correctly (allow a multiplicative constant error) (ignore \int signs)
	$\frac{1}{0.08} \ln(10 + 0.04v^2) = -x + c$ oe	A1 A1	Correct indefinite integrals, ignore c /limits here
	$x = 0, v = u \Rightarrow c = \frac{1}{0.08} \ln(10 + 0.04u^2)$	M1	Find c in terms of u
	$x = \frac{1}{0.08} \ln\left(\frac{10 + 0.04u^2}{10 + 0.04v^2}\right)$	A1	Correct formula for v and x , aef
	$v = 0, x = \frac{1}{0.08} \ln(1 + 0.004u^2)$	M1	Substitute $v = 0$ and solve for u
	$x = 50, u = \sqrt{\frac{(e^4 - 1)}{0.004}} = 116 \text{ ms}^{-1}$	A1	awrt 116 ms^{-1} .
Alternative I	$x = 50, v = 0 \Rightarrow c = \frac{1}{0.08} \ln(10) + 50$	M1	Find numerical c
	$x = 50 + \frac{1}{0.08} \ln\left(\frac{10}{10 + 0.04v^2}\right)$	A1	Correct formula for v and x , aef
	$x = 0, v = u, 50 = \frac{1}{0.08} \ln(1 + 0.004u^2)$	M1	Substitute $x = 0, v = u$ and solve for u
	$u = \sqrt{\frac{(e^4 - 1)}{0.004}} = 116 \text{ ms}^{-1}$	A1	awrt 116 ms^{-1} .

Question	Answer	Marks	Notes
12(ii)	Alternative II		
	$\left[\frac{1}{0.08} \ln(10 + 0.04v^2) \right]_u^0 = [-x]_0^{50} = -50$	B1	Correct limits and evaluation on x side
	$\ln(10) - \ln(10 + 0.04u^2)$ or $\ln\left(\frac{10}{10 + 0.04u^2}\right)$ oe	M1	Correctly dealing with limits on v side
	$\frac{1}{0.08}(\ln(10) - \ln(10 + 0.04u^2)) = -50$	M1	Solve for u
	$u = \sqrt{\frac{(e^4 - 1)}{0.004}} = 116 \text{ ms}^{-1}$	A1	awrt 116 ms^{-1} .
13(i)	At top, $mg (+ T) = mv^2/r$	M1	N2(↓) at top
	so $v^2 \geq 0.6g$ or 6	M1A1	Obtain inequality (or equation); correct, a.e.f. [No mention of T: M1M0A1]
	C of E: $\frac{1}{2}mv^2 + 2mgr = \frac{1}{2}mu^2$	M1A1	Use conservation of energy; correct equation
	$v^2 = u^2 - 4gr$; $u^2 \geq 5gr = 30$,	M1	Solve for u
	$\Rightarrow u_{\min} = 5.48$	A1	Answer, awrt 5.48. Condone $\sqrt{30}$ Common Error: $v_{\min} = 0$ at top leading to $u_{\min} = \sqrt{24} = 4.90$. M0M0A0M1A1M1A0 or SC3.
13(ii)	$mg \cos\theta + T = mv^2/r$	M1	Resolve inwards at general angle, T not needed (NB $-mg \cos\theta$ if with downward vertical). Condone sign errors or v^2/r for M1
	$v^2 = gr \cos\theta$	A1	Correct condition
	$\frac{1}{2}mv^2 = \frac{1}{2}m5^2 - 0.6mg(1 + \cos\theta)$	M1A1	C of E for general angle; correct equation. $-\cos\theta$ if with downward vertical. Condone mgh for M1.
	$u^2 = gr(2 + 3 \cos\theta)$ [25 = 6(2 + 3cosθ)]	M1	Find value for $\cos\theta$ [= ±13/18]
	$\theta = \cos^{-1}(0.7222) = 43.8^\circ$	A1	Answer in range [43.7. 43.8]
14(i)	$0.05\ddot{x} = -\frac{0.6}{1.2}x$ or $\ddot{x} = -10x$	M1	Use $ma = -\lambda x/l$. If two non-zero tensions, M0
		A1	Condone $a = -10x$

Question	Answer	Marks	Notes
14(ii)	$v = \omega\sqrt{(a^2 - x^2)} = \omega a$	M1	Use $v = \omega\sqrt{(a^2 - x^2)}$
	$= 0.316$	A1	$\omega = \sqrt{10}$ and $a = 0.1 \Rightarrow v = \sqrt{10}/10$ or awrt 0.316
	Alternative I $\frac{1}{2} 0.05v^2 = 0.6 \times 0.1^2 / (2 \times 1.2)$	M1	Use C of E: $\frac{1}{2} mv^2 = \lambda x^2 / 2l$
	$\Rightarrow v = 0.316$	A1	$x = 0.1$ and given values $\Rightarrow v =$ awrt 0.316
	Alternative II $x = 0.1 \cos(t/\sqrt{10})$	M1	Solving SHM equation with initial conditions
	$\dot{x} = -0.1\sqrt{10} \sin(\sqrt{10}t) \Rightarrow v = \pm 0.316$	A1	Differentiating and finding max $\Rightarrow v =$ awrt 0.316
14(iii)	One complete period: $2\pi/\omega$	M1	Use $2\pi/\omega$ somewhere
	$= 1.987$ s	A1	Correct time for SHM parts. Allow $2\pi/\sqrt{10}$
	2×0.4 m at constant speed v :	M1	Or one way: 0.4 m at constant speed v
	2.53 s or $0.8\sqrt{10}$ o.e.	A1	1.26s
	Total 4.52	A1	Allow $\frac{\sqrt{10}(4 + \pi)}{5}$ oe